Lecture 14

Example

```
n = A.length                          O(w)
let P[1...n] be a new array           O(z)
for i = 1 to n
    P[i] = Random(1, n^3)             O(y)
sort A, using P as sort keys          O(x)
```

The complexity is $T(n) = O(w + z + nY + x)$

Example

```
i = 1                                 1
while (i ≤ n and x ≠ a)               # of iterations unknown?
    i = i + 1                         1
if i ≤ n then location = i
else location = 0
return location {i if x = a_i, or is 0 if x is not found}     1
```

★ if $a_i = x$, then we have $(i-1)$ iterations

$$T(n) = 1 + \left[\sum_{k=1}^{i-1} (1)\right] + 1 \approx 1 + \left[\sum_{k=1}^{i-1} 1\right]$$

This is $O(n)$

Back to Average - case complexity of linear search

- Assume:

    1. The searched element x is in the list
    2. The distribution of x's position follows "uniform distribution"

    → I.e. x could be anywhere in the list with the same probabilities

    → Domain: x's position, Distribution: $Pr(x = a_i) = \frac{1}{n}$

$$avg_{comparison} = \sum_{i=1}^{n} i\left(\frac{1}{n}\right) = (1)\left(\frac{1}{n}\right) + (2)\left(\frac{1}{n}\right) + (3)\left(\frac{1}{n}\right) + \cdots + (n-1)\frac{1}{n} + n\left(\frac{1}{n}\right)$$

So...

- Technically, if we simply say
    - "Average - case complexity of linear search is $\Theta(n)$"
- This statement is incomplete
- The average also depends on the input distribution

Mathematical Induction

- The induction is nothing more than...
  - An infinite loop of "Modus Ponens" $\longrightarrow \left\{ \begin{array}{l} P \\ P \to q \\ \hline q \end{array} \right.$

Example

- Prove that $1^3 + 2^3 + 3^3 + \dots + (n)^3 = \dfrac{n^2(n-1)^3}{4}$

Base Case: $1^3 \overset{???}{=} \dfrac{1^2(1+1)^2}{4}$

Left side: $1^3 = 1$

Right side: $\dfrac{1^2(1+1)^2}{4} = 1$

so $1 = 1$ so base case works

Inductive hypothesis: $P(k)$ is true

$$1^3 + 2^3 + 3^3 + \dots + k^3 = \frac{k^2(k+1)^2}{4}$$

Inductive Step: $P(k) \to P(k+1)$

$$P(k+1):\ 1^3 + 2^3 + 3^3 + \dots + k^3 + (k+1)^3 \overset{???}{=} \frac{(k+1)^2(k+2)^2}{4}$$

Left-Hand-Side: $\underbrace{\dfrac{k^2(k+1)^2}{4}}_{\text{Inductive hypothesis}} + (k+1)^3$

Right-Hand-Side: $\dfrac{(k+1)^2(k+2)^2}{4}$

Now show left side = right side

Hueristics

- If the proof is LHS = RHS, we almost have a procedure.
    - Write down LHS and RHS of P(k+1)
        - * Identify P(k) from either LHS or RHS
        - * Use P(k) to substitute part of LHS or RHS
        - * Then, try to reach from LHS to RHS (or RHS to LHS)